OCR
Oxford Cambridge and RSA

Wednesday 14 June 2023 – Afternoon

AS Level Further Mathematics A

Y533/01 Mechanics

Time allowed: 1 hour 15 minutes

9975335891

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for AS Level Further Mathematics A
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

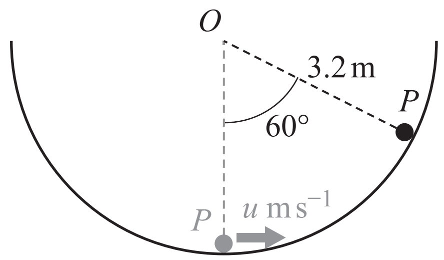
ADVICE
- Read each question carefully before you start your answer.

1 Two particles A, of mass m kg, and B, of mass $3m$ kg, are connected by a light inextensible string and placed together at rest on a smooth horizontal surface with the string slack. A is projected along the surface, directly away from B, with a speed of $2.4\,\mathrm{m\,s^{-1}}$.

(a) Find the speed of B immediately after the string becomes taut. **[2]**

(b) Find, in terms of m, the magnitude of the impulse exerted on B as a result of the string becoming taut. **[2]**

2



A small body P of mass 3 kg is at rest at the lowest point of the inside of a smooth hemispherical shell of radius 3.2 m and centre O.

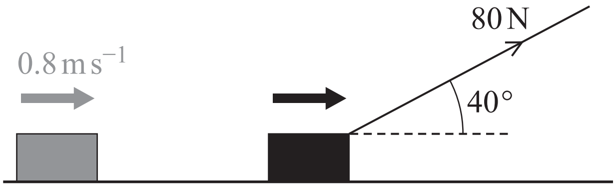
P is projected horizontally with a speed of $u\,\mathrm{m\,s^{-1}}$. When P first comes to instantaneous rest OP makes an angle of 60° with the downward vertical through O.

(a) Find the value of u. **[4]**

(b) State **one** assumption made in modelling the motion of P. **[1]**

3 A crate of mass 45 kg is sliding with a speed of $0.8\,\mathrm{m\,s^{-1}}$ in a straight line across a smooth horizontal floor. One end of a light inextensible rope is attached to the crate. At a certain instant a builder takes the other end of the rope and starts to pull, applying a constant force of 80 N for 5 seconds.

While the builder is pulling the crate, the rope makes a constant angle of 40° above the horizontal. Both the rope and the velocity of the crate lie in the same vertical plane (see diagram).



It may be assumed that there is no resistance to the motion of the crate.

(a) Determine the work done by the builder in pulling the crate. **[4]**

(b) **(i)** Find the kinetic energy of the crate at the instant when the builder stops pulling the crate. **[2]**

(ii) Explain why the answers to part **(a)** and part **(b)(i)** are **not** equal. **[1]**

(c) Find the average power developed by the builder in pulling the crate. **[1]**

(d) Calculate the total impulse exerted on the crate by the builder. **[2]**

4 A rower is rowing a boat in a straight line across a lake. The combined mass of the rower, boat and oars is 240 kg. The maximum power that the rower can generate is 450 W.

In a model of the motion of the boat it is assumed that the total resistance to the motion of the boat is 150 N at any instant when the boat is in motion.

(a) Find the maximum possible acceleration of the boat, according to the model, at an instant when its speed is $0.5\,\mathrm{m\,s^{-1}}$. **[2]**

At one stage in its motion the boat is travelling at a constant speed and the rower is generating power at an average rate of 210 W, which is assumed to be constant. The boat passes a pole and then, after travelling 350 m, a second pole.

(b) Determine how long it takes, according to the model, for the boat to travel between the two poles. **[4]**

(c) State a reason why the assumption that the rower's generated power is constant may be unrealistic. **[1]**

5 Two identical spheres, A and B, each of mass 4 kg, are moving directly towards each other along the same straight line on a smooth horizontal surface until they collide. Before they collide, the speeds of A and B are $5\,\mathrm{m\,s^{-1}}$ and $3\,\mathrm{m\,s^{-1}}$ respectively. Immediately after they collide, the speed of A is $2\,\mathrm{m\,s^{-1}}$ and its direction of motion has been reversed.

(a) **(i)** Determine the velocity of B immediately after A and B collide. **[3]**

(ii) Show that the coefficient of restitution between A and B is $\frac{3}{4}$. **[2]**

(iii) Calculate the total loss of kinetic energy due to this collision. **[2]**

Sphere B goes on to strike a fixed wall directly. As a result of this collision B moves along the same straight line with a speed of $4\,\mathrm{m\,s^{-1}}$.

(b) Find the coefficient of restitution between B and the wall, stating whether the collision between B and the wall is perfectly elastic. **[2]**

(c) Determine the magnitude of the impulse that B exerts on A the next time that they collide. **[5]**

6 The physical quantity pressure, denoted by P, can be calculated using the formula $P = \frac{F}{A}$ where F is a force and A is an area.

(a) Find the dimensions of P. **[1]**

An object of mass m is moving on a smooth horizontal surface subject to a system of forces which begin to act at time $t = 0$. The initial velocity of the object is u and its velocity and acceleration at time t are denoted by v and a respectively. The object exerts a pressure P on the surface. The total work done by the forces is denoted by W.

A Mathematics class suggests three formulae to model the quantity W.

The first suggested formula is $W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 + mP$.

(b) Use dimensional analysis to show that this formula cannot be correct. **[2]**

The second suggested formula is $W = ku^{\alpha}v^{\beta}t^{\gamma}$ where k is a dimensionless constant.

(c) Use dimensional analysis to show that this formula cannot be correct for any values of α, β and γ. **[2]**

The third suggested formula is $W = ku^{\alpha}a^{\beta}m^{\gamma}t^{\delta}$ where k is a dimensionless constant.

(d) **(i)** Explain why it is **not** possible to use dimensional analysis to determine the values of α, β, γ and δ for the third suggested formula. **[1]**

(ii) Given that $\alpha = 3$, use dimensional analysis to determine the values of β, γ and δ for the third suggested formula. **[3]**

(iii) By considering what the formula predicts for large values of t, explain why the formula derived in part **(d)(ii)** is likely to be incorrect. **[1]**

7 Two identical light, inextensible strings S_1 and S_2 are each of length 5 m. Two identical particles P and Q are each of mass 1.5 kg.

One end of S_1 is attached to P. The other end of S_1 is attached to a fixed point A on a smooth horizontal plane. P moves with constant speed in a horizontal circular path with A as its centre (see **Fig. 1**).

One end of S_2 is attached to Q. The other end of S_2 is attached to a fixed point B. Q moves with constant speed in a horizontal circular path around a point O which is vertically below B. At any instant, BQ makes an angle of θ with the downward vertical through B (see **Fig. 2**).

Fig. 1

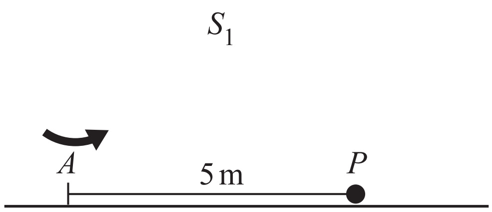
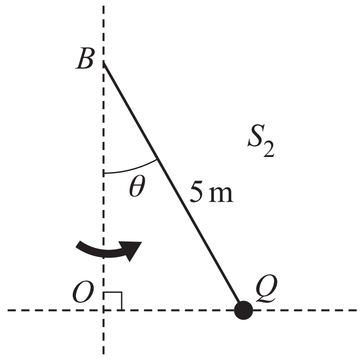


Fig. 2



(a) Given that the angular speed of P is the same as the angular speed of Q, show that the tensions in S_1 and S_2 have the same magnitude. **[3]**

(b) You are given instead that the kinetic energy of P is 39.2 J and that this is the same as the kinetic energy of Q.

Determine the difference between the times taken by P and Q to complete one revolution. Give your answer in an exact form. **[7]**

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